ANALYSIS OF WAVEGUIDES WITH METAL INSERTS

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ABSTRACT

A systematic analysis of waveguides with metal inserts is presented. The method is based on a field expansion in terms of the normal modes of the corresponding hollow waveguide without metal inserts. The analysis leads to two main formulations: the matrix formulation and the moment method formulation. The matrix formulation is suitable for structures with smooth metal inserts, which are free from sharp edges, while the moment method is more suitable for metal sheets (e.g. strips and fins) or metal inserts with sharp edges (e.g. ridges).

The method is applied to the analysis of ridged waveguides and finlines, and leads to a generalization of the widely used spectral domain technique with respect that ridges, fins, and strips with finite thickness can now equally be analyzed. Any existing routine for the analysis of planar structures, which is based on the spectral domain technique, can slightly be modified in order to take the metallization thickness into account.

INTRODUCTION

Most of the existing methods for the analysis of guiding structures depend to a great extent on the specific geometry of the individual structure. A systematic method for the analysis of waveguides with arbitrarily shaped cross section is strongly recommended for computer-aided design and optimization of complex microwave systems, in which different guiding structures are involved. Although the finite element (or difference) method (e.g. [1], [2]) and the mode matching technique (e.g. [3], [4]) are capable of analyzing a wide variety of structures, their storage and/or CPU time requirement represent severe restrictions on any economical CAD algorithm.

Many guiding structures simply have a rectangular or a circular outer boundary and one (or more) touching or non-touching metal inserts which may be either solid or sheet-like. Examples are striplines and microstrips, finned waveguides and finlines, ridged waveguides and multi-conductor transmission lines. The electromagnetic field inside these structures can be expanded in terms of the eigenmodes of the corresponding hollow rectangular or circular waveguide such that the different expansions vanish everywhere inside the metal inserts. This procedure corresponds to the one-dimensional Fourier series, in which a function which vanishes over a certain interval can be expanded with respect to the harmonics corresponding to a larger interval which includs the smaller one.

In this paper, only homogeneously filled waveguides will be considered. If the method presented here is, however, combined with that presented in [5], inhomogeneously filled structures can equally be analyzed.

BASIC FORMULATION

Fig. 1 shows the cross section of a waveguide with a single, touching or non-touching metal insert. Extending the anlysis to multi-conductor transmission lines is straight-forward. The direction of propagation, in which the structure is uniform, is taken along the z-axis with corresponding propagation constant β. Let $\{h_{zn}\}$ and $\{e_{zn}\}$ be the complete sets of axial magnetic and electric fields which characterize the TE and TM modes, respectively, of the hollow waveguide (i.e. with the metal insert S_0 removed). h_{zn} and e_{zn} are real functions of the transverse coordinates, which correspond to cutoff wave numbers k_{nh} and k_{ne}, respectively, and satisfy the following orthogonality relations [6]:

$$\int_S h_{zn}\, h_{zm}\, dS = P_{nh}\, \delta_{nm},$$

$$\int_S e_{zn}\, e_{zm}\, dS = P_{ne}\, \delta_{nm}. \qquad (1)$$

Because the structure is homogeneously filled (empty), it can support either TE or TM modes (the TEM mode will be shown to be a TM mode with vanishing cutoff wave number). The completeness property of $\{h_{zn}\}$ and $\{e_{zn}\}$ can now be used in order to express the different field components in the original structure (with the metal insert S_0 present) as expansions in terms of $\{h_{zn}\}$ and $\{e_{zn}\}$ which vanish everywhere over S_0.

Let $\underline{e}$, $\underline{h}$, e_z and h_z be the transverse electric, transverse magnetic, axial electric, and axial magnetic field, respectively, with the z-dependence $e^{-j\beta z}$ being dropped out, then

$$\underline{\nabla}_t \times \underline{e} = -j\omega\mu_o h_z \hat{\underline{k}},$$

$$\underline{\nabla}_t \times \underline{h} = j\omega\varepsilon_o e_z \hat{\underline{k}},$$

$$\underline{\nabla}_t h_z + j\beta \underline{h} = j\omega\varepsilon_o (\hat{\underline{k}} \times \underline{e}),$$

$$\underline{\nabla}_t e_z + j\beta \underline{e} = -j\omega\mu_o (\hat{\underline{k}} \times \underline{h}), \qquad (2)$$

where $\underline{\nabla}_t$ is the transverse del-operator and $\hat{\underline{k}}$ is the unit vector in axial direction.

A) TE modes

Because the tangential component of $\underline{h}$ has a step discontinuity at C_o, $(\underline{\nabla}_t \times \underline{h})$, which includes the normal derivative of the tangential component, behaves as a dirac-delta function there. This dirac-delta function is just the axial component of the surface current at C_o. $(\underline{\nabla}_t \times \underline{h})$ can then vanish everywhere over S except at C_o, and hence, $\underline{h}$ cannot be expanded in terms of the curl-free set $\{\underline{\nabla}_t h_{zn}\}$ only. It needs, in addition, the divergence-free set $\{\underline{\nabla}_t e_{zn} \times \hat{\underline{k}}\}$. The transverse components $\underline{h}$ and $\underline{e}$ can then be expanded as

$$\underline{h} = -j\beta \left[\sum_n \frac{a_n^{(h)}}{\sqrt{P_{nh}}} \underline{\nabla}_t h_{zn} + \sum_n \frac{b_n^{(h)}}{\sqrt{P_{ne}}} \left(\underline{\nabla}_t e_{zn} \times \hat{\underline{k}}\right) \right],$$

$$\underline{e} = \frac{j\omega\mu_o}{j\beta} \left(\underline{h} \times \hat{\underline{k}}\right). \qquad (3)$$

Because the normal component of $\underline{h}$ is continuous across C_o, h_z can be obtained from $(j\beta h_z = \underline{\nabla}_t \cdot \underline{h})$ through a term-by-term differentiation of (3):

$$h_z = \sum_n \frac{a_n^{(h)}}{\sqrt{P_{nh}}} k_{nh}^2 h_{zn}. \qquad (4)$$

The expansion coefficients $a_n^{(h)}$ and $b_n^{(h)}$ are obtained by making use of the orthogonality relations (1):

$$a_n^{(h)} = \frac{-1}{j\beta k_{nh}^2 \sqrt{P_{nh}}} \int_{S-S_o} \underline{h} \cdot \underline{\nabla}_t h_{zn} \, dS,$$

$$b_n^{(h)} = \frac{-1}{j\beta k_{ne}^2 \sqrt{P_{ne}}} \int_{S-S_o} \underline{h} \cdot \left(\underline{\nabla}_t e_{zn} \times \hat{\underline{k}}\right) dS. \qquad (5)$$

The integration is taken over $(S-S_o)$ because $\underline{h}$ vanishes everywhere on S_o. After some mathematical manipulations, one obtains

$$a_n^{(h)} = \frac{1}{\left(k_{nh}^2 - k_c^2\right) k_{nh}^2 \sqrt{P_{nh}}} \oint_{C_o} h_z \left(\hat{\underline{n}} \cdot \underline{\nabla}_t h_{zn}\right) d\ell,$$

$$b_n^{(h)} = \frac{1}{k_{ne}^2 k_c^2 \sqrt{P_{ne}}} \oint_{C_o} h_z \left[(\hat{\underline{n}} \times \hat{\underline{k}}) \cdot \underline{\nabla}_t e_{zn}\right] d\ell, \qquad (6)$$

where k_c is the cutoff wave number $(k_c^2 = k_0^2 - \beta^2)$.

B) TM modes

Because the tangential component of $\underline{e}$ is continuous across C_o, $(\underline{\nabla}_t \times \underline{e})$ is free from dirac-delta functions at C_o and hence can vanish everywhere on S. $\underline{e}$ can consequently be expanded in terms of the curl-free set $\{\underline{\nabla}_t e_{zn}\}$. The transverse components $\underline{e}$ and $\underline{h}$ can then be written as

$$\underline{e} = \sum_n \frac{a_n^{(e)}}{\sqrt{P_{ne}}} \underline{\nabla}_t e_{zn},$$

$$\underline{h} = \frac{j\omega\varepsilon_o}{j\beta} \left(\hat{\underline{k}} \times \underline{e}\right). \qquad (7)$$

Because e_z is continuous across C_o, it can be obtained from $(-j\beta \underline{\nabla}_t e_z = k_c^2 \underline{e})$ through a term-by-term integration of (7):

$$e_z = \frac{-k_c^2}{j\beta} \sum_n \frac{a_n^{(e)}}{\sqrt{P_{ne}}} e_{zn}. \qquad (8)$$

Following a procedure similar to that for the TE modes, the expansion coefficients $a_n^{(e)}$ are given by

$$a_n^{(e)} = \frac{1}{\left(k_c^2 - k_{ne}^2\right) \sqrt{P_{ne}}} \oint_{C_o} e_{zn} \left(\hat{\underline{n}} \cdot \underline{e}\right) d\ell. \qquad (9)$$

If $k_c = 0$, $e_z = 0$ and the TEM mode is obtained. The quantity $(j\beta e_z/k_c^2)$ is, however, finite and plays the same role as an electrostatic potential φ.

MATRIX FORMULATION

If the contour C_o is smooth enough, i.e. free from sharp edges, the different field components are regular at C_o and hence their series representations converge rapidly. In this case, h_z in (6) and $(\hat{\underline{n}} \cdot \underline{e})$ in (9) can be replaced by ***twice*** their series representations (4) and (7), respectively. The factor ***two*** is due to the step discontinuities of h_z and $(\hat{\underline{n}} \cdot \underline{e})$ at C_o. The series in (4) and (7) converge then to only half the value of h_z and $(\hat{\underline{n}} \cdot \underline{e})$, respectively, at C_o.

Substituting (4) into (6), one arrives at the following matrix eigenvalue equation for TE modes

$$\left(\left[\Lambda^h\right] - \left[C^h\right] \right) \left[\Lambda^h\right] \underline{a}^{(h)} = k_c^2 \left[\Lambda^h\right] \underline{a}^{(h)}, \qquad (10)$$

where $[\Lambda^h]$ is a diagonal matrix with elements k_{nh}^2, $\underline{a}^{(h)}$ is a column vector with elements $a_n^{(h)}$. The elements of the square matrix $[C^h]$ are given by

$$C_{nm}^h = \frac{2}{\sqrt{P_{nh} P_{mh}}} \oint_{C_o} h_{zm} \left(\hat{\underline{n}} \cdot \underline{\nabla}_t h_{zn}\right) d\ell. \qquad (11)$$

Substituting (7) into (9), the following matrix eigenvalue equation is obtained for TM modes:

$$\left(\left[\Lambda^e\right] + \left[C^e\right] \right) \underline{a}^{(e)} = k_c^2 \underline{a}^{(e)}, \qquad (12)$$

where $[\Lambda^e]$ is a diagonal matrix with elements k_{ne}^2, $\underline{a}^{(e)}$ is a column vector with elements $a_n^{(e)}$. The elements of the square matrix $[C^e]$ are now given by

$$C^{e}_{nm} = \frac{2}{\sqrt{P_{ne}\, P_{me}}} \oint_{C_0} e_{zn} \left(\hat{\underline{n}} \cdot \underline{\nabla}_t e_{zm}\right) d\ell. \qquad (13)$$

The matrix eigenvalue equations (10) and (12) have essentially doubly infinite order. For computational purposes, the series in (3), (4), (7) and (8) must be truncated, retaining a finite number of terms. The matrices in (10) and (12) then become of finite order.

MOMENT METHOD FORMULATION

If the contour C_0 is open (i.e. the metal insert is infinitesimally thin) or has sharp edges, some field components become singular at the edges. A large number of terms must then be retained in the series representing these components, which results in oversized matrices in (10) and (12). For these cases, it is more suitable to expand h_z and $(\hat{\underline{n}}\cdot\underline{e})$ at C_0 in terms of basis functions, which individually satisfy the edge conditions:

$$h_z\Big|_{C_0} = \sum_i I_i\, \eta_i,$$

$$\left(\hat{\underline{n}} \cdot \underline{e}\right)\Big|_{C_0} = \sum_i V_i\, \xi_i. \qquad (14)$$

Here I_i and V_i are expansion coefficients, and η_i and ξ_i are the basis functions. The expansion coefficients are then determined by asking for a vanishing tangential electric field (or normal magnetic field) at C_0. If we test the vanishing fields at C_0 by the same basis functions (Galerkin's procedure) one arrives at the following equations:

$$\left\{ k_c^2 \left[\tilde{C}^{hh}\right]^t \left(k_c^2 \left[I\right] - \left[\Lambda^h\right]\right)^{-1} \left[\tilde{C}^{hh}\right] + \left[\tilde{C}^{he}\right]^t \left[\tilde{C}^{he}\right] \right\} \underline{I} = 0, \qquad (15)$$

for TE modes, and

$$\left[\tilde{C}^{ee}\right]^t \left(k_c^2 \left[I\right] - \left[\Lambda^e\right]\right)^{-1} \left[\tilde{C}^{ee}\right] \underline{V} = 0, \qquad (16)$$

for TM modes. $\underline{I}$ and $\underline{V}$ are column vectors with elements I_i and V_i, respectively, and the elements of the matrices $[\tilde{C}^{hh}]$, $[\tilde{C}^{he}]$ and $[\tilde{C}^{ee}]$ are given by

$$\tilde{C}^{hh}_{ni} = \frac{1}{k_{nh}\sqrt{P_{nh}}} \oint_{C_0} \eta_i \left(\hat{\underline{n}} \cdot \underline{\nabla}_t h_{zn}\right) d\ell,$$

$$\tilde{C}^{he}_{ni} = \frac{1}{k_{ne}\sqrt{P_{ne}}} \oint_{C_0} \eta_i \left((\hat{\underline{n}} \times \hat{\underline{k}}) \cdot \underline{\nabla}_t e_{zn}\right) d\ell,$$

$$\tilde{C}^{ee}_{ni} = \frac{1}{\sqrt{P_{ne}}} \oint_{C_0} \xi_i\, e_{zn}\, d\ell. \qquad (17)$$

The size of the characteristic matrix in (15) and (16) is equal to the number of basis functions used, which can be greatly reduced if the basis functions are properly chosen. On the other hand, each matrix element is a doubly infinite sum, which must be truncated if it is not expressible in closed form. The truncation limit can, however, be put sufficiently high in order to correctly account for the singular behaviour of some field components at the edges. In this procedure the truncation limit does not influence the size of the characteristic matrix.

The moment method formulation is basically the same as that presented in [7], except for the form of the Green's functions used. In [7], the singular parts of the Green's functions have been written explicitly as logarithmic functions. The singularity is then removed by integration when the actual field is calculated. In this formulation, the fields themselves and not the Green's functions are expressed. No singularity is then included, and all series converge uniformly.

APPLICATION TO RIDGED WAVEGUIDE

Consider the ridged waveguide shown in Fig. 2. For simplicity, only TM modes with electric wall symmetry at x = 0 will be analyzed:

$$e_{znm} = \sin\frac{n\pi x}{a} \cdot \sin\frac{m\pi y}{b},$$

$$k^2_{nme} = \left(\frac{n\pi}{a}\right)^2 + \left(\frac{m\pi}{b}\right)^2,$$

$$P_{nme} = \frac{ab}{4}. \qquad (18)$$

The normal electric field at the ridge is expanded as

$$\left(\hat{\underline{n}} \cdot \underline{e}\right)\Big|_{C_0} = \begin{cases} -e_y\Big|_{y_0} = \sum_i V_i^{(1)} \xi_i^{(1)}(x) & 0 \le x \le x_0, \\ +e_x\Big|_{x_0} = \sum_i V_i^{(2)} \xi_i^{(2)}(y) & y_0 \le y \le b. \end{cases} \qquad (19)$$

Equation (16) can then be written as

$$\begin{bmatrix} \left[Y^{(11)}\right] & \left[Y^{(12)}\right] \\ \left[Y^{(21)}\right] & \left[Y^{(22)}\right] \end{bmatrix} \begin{bmatrix} \underline{V}^{(1)} \\ \underline{V}^{(2)} \end{bmatrix} = 0, \qquad (20)$$

where $\underline{V}^{(1)}$ and $\underline{V}^{(2)}$ are column vectors with elements $V_i^{(1)}$ and $V_i^{(2)}$, respectively. The elements of the Y-matrices are given by

$$Y^{(11)}_{ij} = \frac{-ab}{2\pi} \sum_{n=1}^{\infty} \frac{\sin \bar{\alpha}_n(\pi-\psi_o)\, \sin \bar{\alpha}_n\psi_o}{\bar{\alpha}_n \sin \bar{\alpha}_n\pi}\, \tilde{\xi}^{(1)}_{in}\, \tilde{\xi}^{(1)}_{jn},$$

$$Y^{(12)}_{ij} = Y^{(21)}_{ji} = \frac{-b^2}{2\pi} \sum_{n=1}^{\infty} \frac{\sin n\varphi_o\, \sin \bar{\alpha}_n\psi_o}{\bar{\alpha}_n \sin \bar{\alpha}_n\pi}\, \tilde{\xi}^{(1)}_{in}\, \tilde{\tilde{\xi}}^{(2)}_{jn}$$

$$= \frac{-a^2}{2\pi} \sum_{m=1}^{\infty} \frac{\sin m\psi_o\, \sin \bar{\gamma}_m(\pi-\varphi_o)}{\bar{\gamma}_m \sin \bar{\gamma}_m\pi}\, \tilde{\tilde{\xi}}^{(1)}_{im}\, \tilde{\xi}^{(2)}_{jm},$$

$$Y^{(22)}_{ij} = \frac{-ab}{2\pi} \sum_{m=1}^{\infty} \frac{\sin \bar{\gamma}_m(\pi-\varphi_o)\, \sin \bar{\gamma}_m\varphi_o}{\bar{\gamma}_m \sin \bar{\gamma}_m\pi}\, \tilde{\xi}^{(2)}_{im}\, \tilde{\xi}^{(2)}_{jm}. \qquad (21)$$

where

$$\varphi_o = \frac{\pi x_o}{a}, \quad \psi_o = \frac{\pi y_o}{b},$$

$$\bar{\alpha}_n = \frac{b}{\pi}\sqrt{k_c^2 - \left(\frac{n\pi}{a}\right)^2}, \quad \bar{\gamma}_m = \frac{a}{\pi}\sqrt{k_c^2 - \left(\frac{m\pi}{b}\right)^2},$$

$$\tilde{\xi}_{in}^{(1)} = \frac{2}{a}\int_0^{x_o} \xi_i^{(1)}(x) \sin\frac{n\pi x}{a}\, dx,$$

$$\tilde{\xi}_{im}^{(2)} = \frac{2}{b}\int_{y_o}^{b} \xi_i^{(2)}(y) \sin\frac{m\pi y}{b}\, dy,$$

$$\tilde{\tilde{\xi}}_{im}^{(1)} = \frac{2}{a}\int_0^{x_o} \xi_i^{(1)}(x) \sin \gamma_m x\, dx,$$

$$\tilde{\tilde{\xi}}_{in}^{(2)} = \frac{2}{b}\int_{y_o}^{b} \xi_i^{(2)}(y) \sin \alpha_n (b-y)\, dy. \quad (22)$$

Referring to e.g. [8], it is easily seen that $[Y^{(11)}]$ is the characteristic matrix of a corresponding structure with single infinitesimally thin strip at $y = y_o$, which extends from $x = 0$ to $x = x_o$, while $[Y^{(22)}]$ is the characteristic matrix of a corresponding structure with single strip at $x = x_o$, which extends from $y = y_o$ to $y = b$. The elements of $[Y^{(12)}]$ are easily obtained from the elements of $[Y^{(11)}]$ by replacing $(a \cdot \sin \bar{\alpha}_n(\pi-\psi_o) \cdot \tilde{\xi}_{jn}^{(1)})$ by $(b \cdot \sin n\varphi_o \cdot \tilde{\tilde{\xi}}_{jn}^{(2)})$.

The formulation presented here consequently is a generalization of the widely used spectral domain technique with respect that the finite metallization thickness is correctly taken into account. Any existing routine for the analysis of planar structures which is based on the spectral domain technique can hence be modified according to the above statements in order to extend its validity to the analysis of finite metallization thickness.

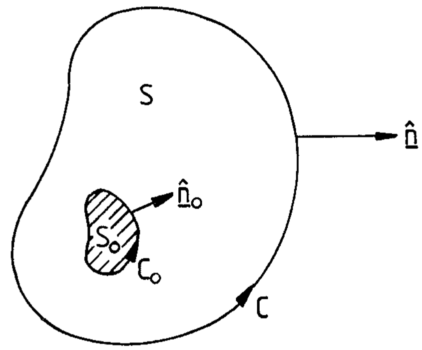

Fig. 1: Cross section of a waveguide with metal insert.

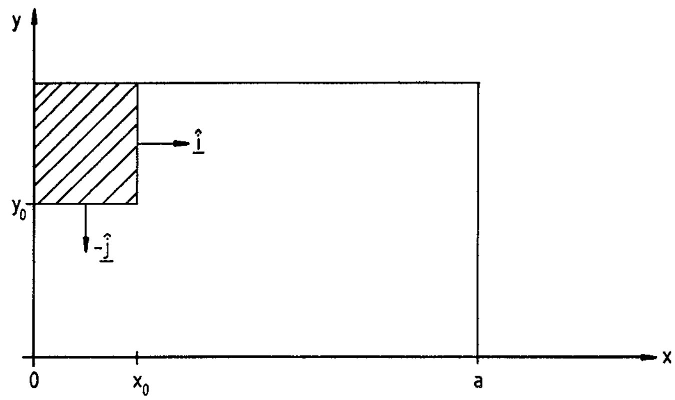

Fig. 2: Cross section of a ridged waveguide.

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